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Stiff structure analogies for analysis of lobed inflatable membrane structures

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Abstract

Analysing inflatable membrane structures poses significant challenges. The negligible bending stiffness causes difficulties for standard approaches such as finite element analysis. Assessing the stability of membrane structures requires examination of the total potential energy of the system. Using analogies with stiff structures provides an alternative basis for analysis of membrane structures. Lobed inflatable structures have analogies with stiff structures such as struts and rings and the stability of the membrane structure can be examined by using the standard analysis methods for stiff structures.

1 Introduction

Characteristics such a low mass and a high ratio of deployed to packed volume make the use of inflatable membrane structures attractive. Negligible bending stiffness is another characteristic of membrane structures and this characteristic makes them difficult to analyse. Assessing stability is particularly difficult and many computational methods fail due to the negligible bending stiffness.

Two methods can provide solutions to the analysis problem. The first method is to use a material model that effectively represents the change from tensile to compressive loads. This method is useful for investigating membrane wrinkling and stress distributions. The second method is to use an analogous structure with bending stiffness (the stiff structure) that behaves similarly to the membrane structure in the area of interest. In this study we examine the structural stability of lobed membrane structures using stiff structures that behave similarly for structural stability.

2 Stability of inflatable structures

Instability is usually associated with structures in compression but inflatable structures with membranes in tension can also become unstable (Calladine [1]). Examining the total potential energy of the system provides a way to understand the instability of inflatable membrane structures. If the other changes in potential energy are small then we need consider only the energy in the inflation gas.

The temperature of a gas is proportional to the volume of the gas for a fixed pressure. This law assumes a closed system but for real systems energy will be lost as volume increases. The configuration of a structure with the largest enclosed volume has the lowest potential energy and is the most stable.

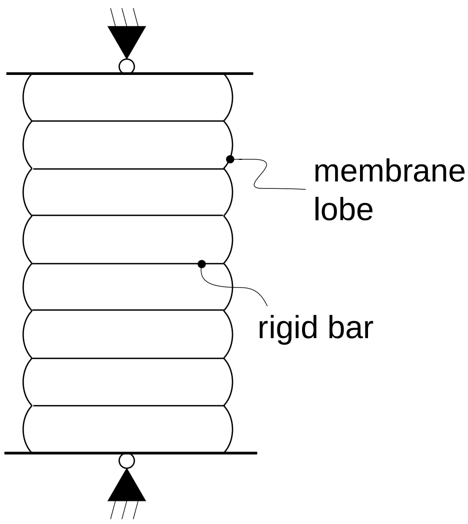


Figure 1: Two-dimensional lobed column

3 Lobed column

Calladine [1] showed that the two-dimensional lobed column of Figure 1 could be unstable for combinations of height, width, number of lobes, and angle subtended by the arc of the lobe. Lennon and Pellegrino [2] extended this work using volume calculations for three-dimensional structures.

A key aspect of Calladine's work was demonstrating the equivalence of the lobed column and an Euler strut. The pressure difference across the membrane causes tension everywhere in the membrane and at first examination it is not obvious that there could be a stability problem. Examining the structural behaviour more closely shows that its structural stability is similar to a column in compression. Figure 2(a) shows the forces for a single lobe with pressure across the membrane balanced by tension in the membrane. Figure 2(b) shows the forces at the intersection between neighbouring lobes. The lobes push against each other at these intersections and the stability behaviour is equivalent to the forces and stiffness of Figure 2(c). We see that the inflated lobes are equivalent to the familiar problem of a column in compression and we can use the mechanics of an Euler strut to calculate the stability of the structure. The key to using this analogy is to examine the structural behaviour of the entire system rather than the material behaviour.

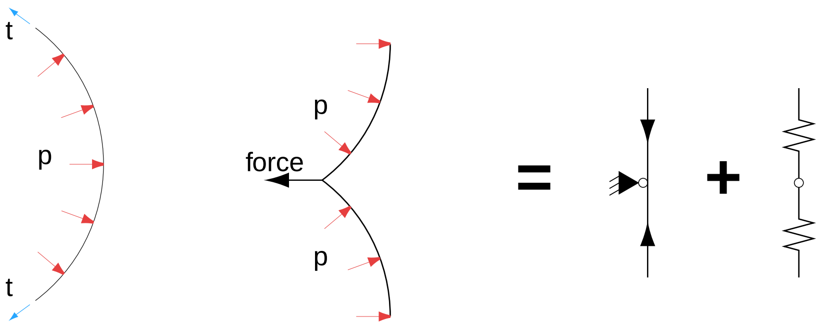
4 Lobed ring

The lobed column of Section 3 shows us an analogy between a stiff structure and an inflatable membrane structure. Buckling for the lobed column is in the plane of the structure and in this section we extend our study to out-of-plane buckling. Figure 3 shows an n-symmetric membrane structure of lobes with a pressure difference across the membrane. Deformation of this lobed ring is constrained by a rigid cylinder. Note that this structure is not real and we are using it only for illustrative purposes.

Figure 4 shows the similarity between this structure and the lobed column problem of Section 3. The equivalent force and stiffness characteristics shown in Figures 4(b) and 4(c) suggest that the stability of the lobed ring is the same as that of a stiff ring buckling under uniform circumferential compression. We can use standard relationships such as those found in Roark [3] to determine the stability of the ring.

5 Lobed isotensoid

Sections 3 and 4 show us that two-dimensional lobed inflatable structures can buckle and demonstrates the analogies with stiff structures. We can also investigate three-dimensional lobed structures such as the lobed isotensoid of Figure 5. Taylor [4] described the isotensoid in a paper in 1919. The isotensoid is a

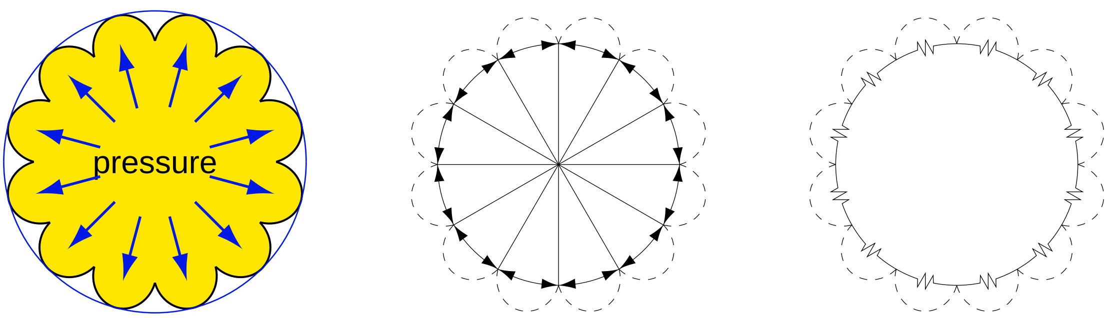


(a) (b) (c)

Figure 2: Structural behaviour of inflated lobes (a) pressure and tension in a lobe (b) forces at lobe intersection (c) equivalent forces and equivalent stiffness

(a) (b)

Figure 3: Restrained lobed ring (a) schematic (b) buckling mode



(a) (b) (c)

Figure 4: Structural behaviour of lobed ring (a) loads on ring (b) equivalent forces (c) equivalent stiffness

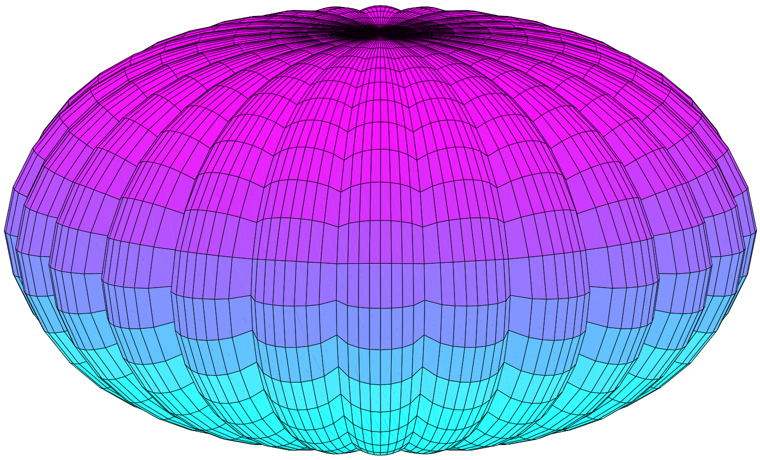


Figure 5: Lobed isotensoid

three-dimension shape that carries tension only in the meridional direction for uniform internal pressure. The hoop stress is zero for this shape. The lobed isotensoid is an adaption of the isotensoid shape that uses meridional load-carrying lines and lobes that transfer hoop tension to the meridional lines. The lobes carry tension only in the hoop direction of the lobe and have no meridional load.

We can find the stiff structure analogy for buckling by examining the forces in the structure. If the lobes can slide along the isotensoid lines then a cross-section at the equator is equivalent to the lobed ring of Section 4. We can think of the isotensoid as a series of rings, infinitesimally thin, stacked on top of each other to give us the structure that we require.

6 Conclusions

- Stability of inflatable structures depends on enclosed area/volume.
- Inflatable structures have analogies with structures that possess bending stiffness (stiff structures).
- Need to consider the lobes as structural members, not just the lobe material stress distribution.
- Can perform our computations on the analogous stiff structure.

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