

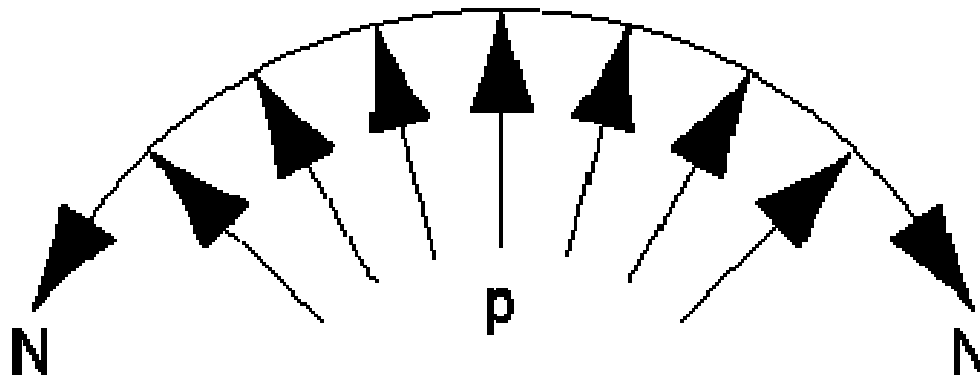
# Geometrical Mechanics for Inflatable Structures (Geometric Structural Mechanics)

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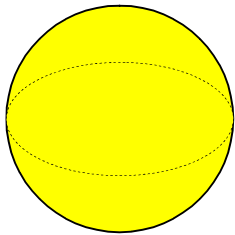
- Geometry and tension
- Reducing tension
- Unidirectional tension
- Some new ideas

# Equilibrium of Inflated Structures



$$N = pr$$

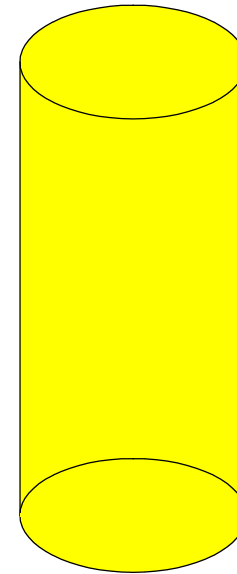
# Sphere and Cylinder



$$\sigma_{sphere} = \frac{pr}{2t}$$

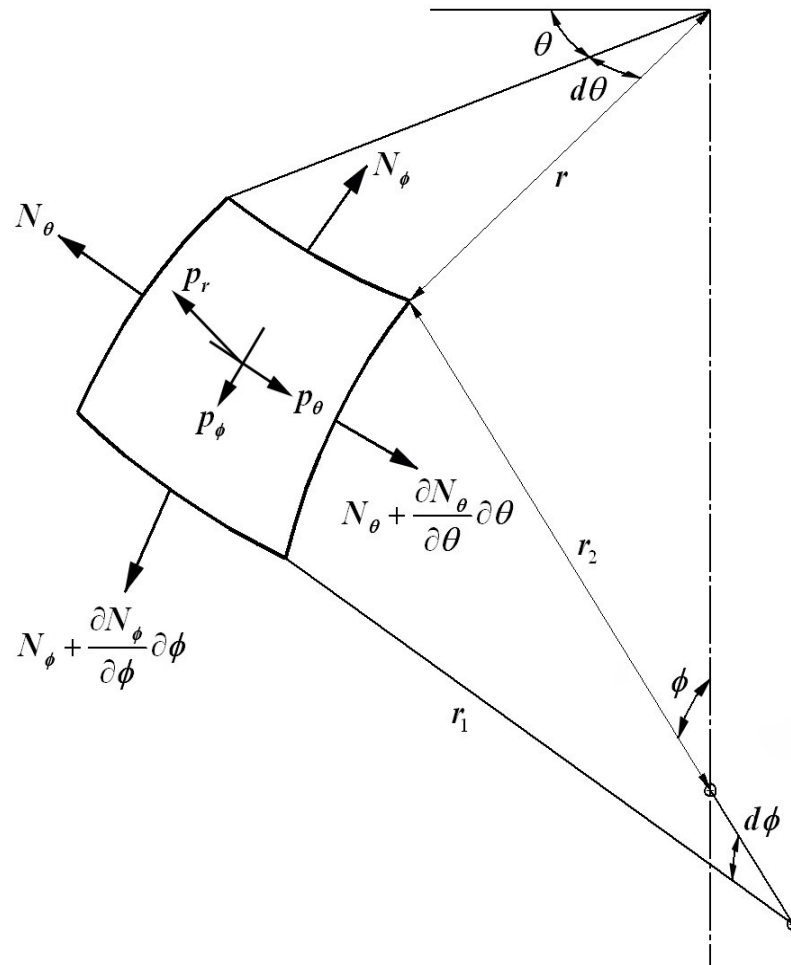
$$\sigma_{cyl,hoop} = \frac{pr}{t}$$

$$\sigma_{cyl,long} = \frac{pr}{2t}$$



maximum cylinder stress = 2 x maximum sphere stress

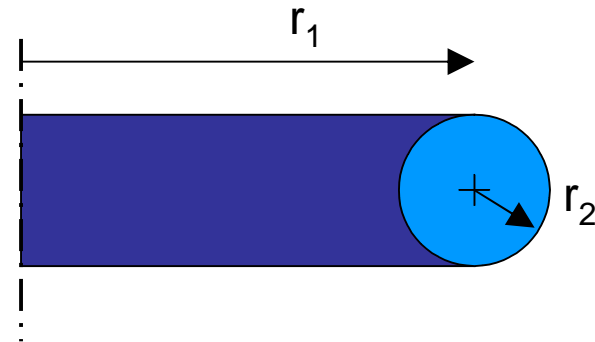
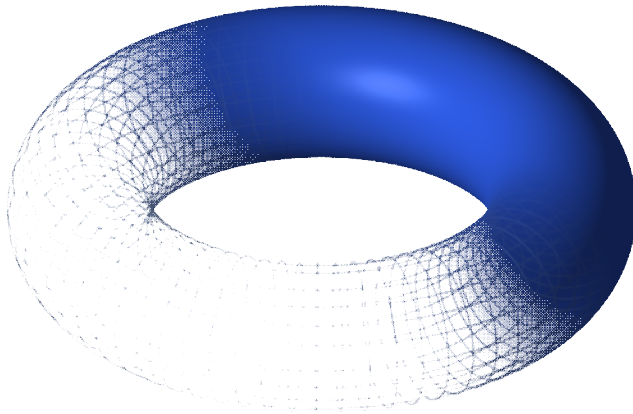
# Generalised Membrane Surface



$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = p$$

(see Flügge 1962 for detail)

# Stress in Inflated Torus



$$\sigma_{\max} = \frac{pr_2}{2t} \cdot \frac{2r_1 - r_2}{r_1 - r_2}$$

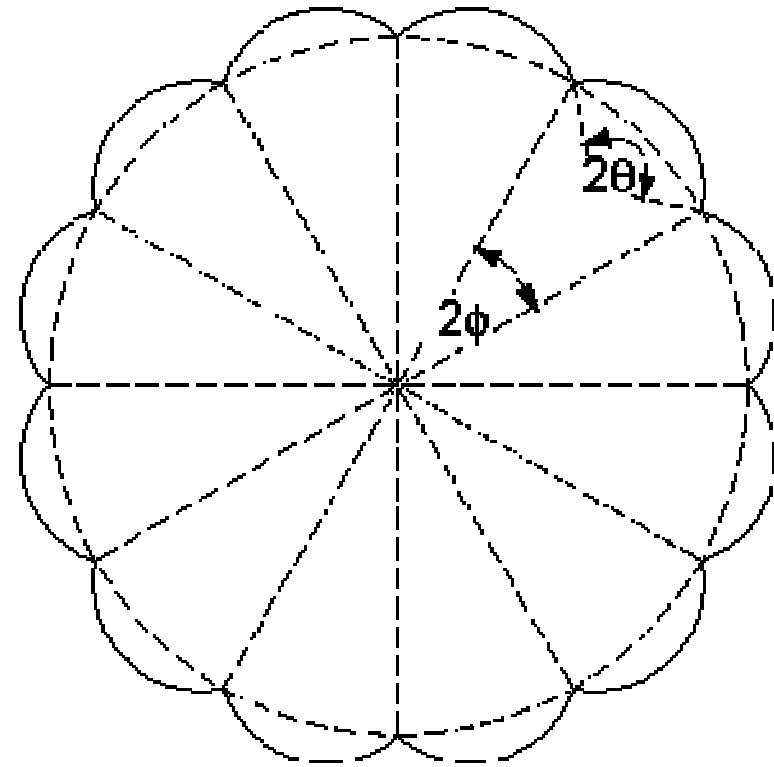
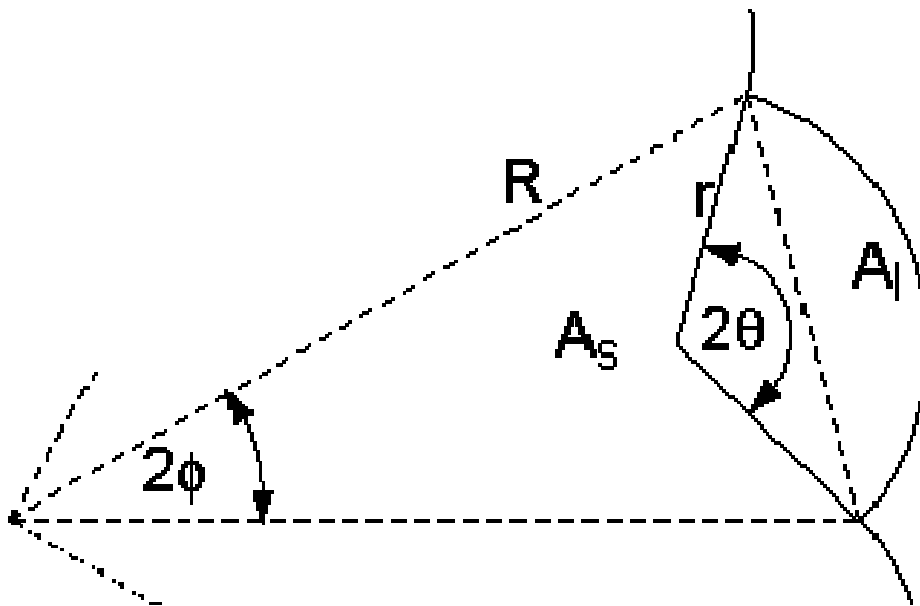
$$r_1 \rightarrow \infty \quad \therefore \quad \frac{2r_1 - r_2}{r_1 - r_2} \rightarrow 2$$

$$\sigma_{\max} \rightarrow \frac{pr_2}{t}$$

(see “Roark’s Formulas for Stress and Strain” for detail)

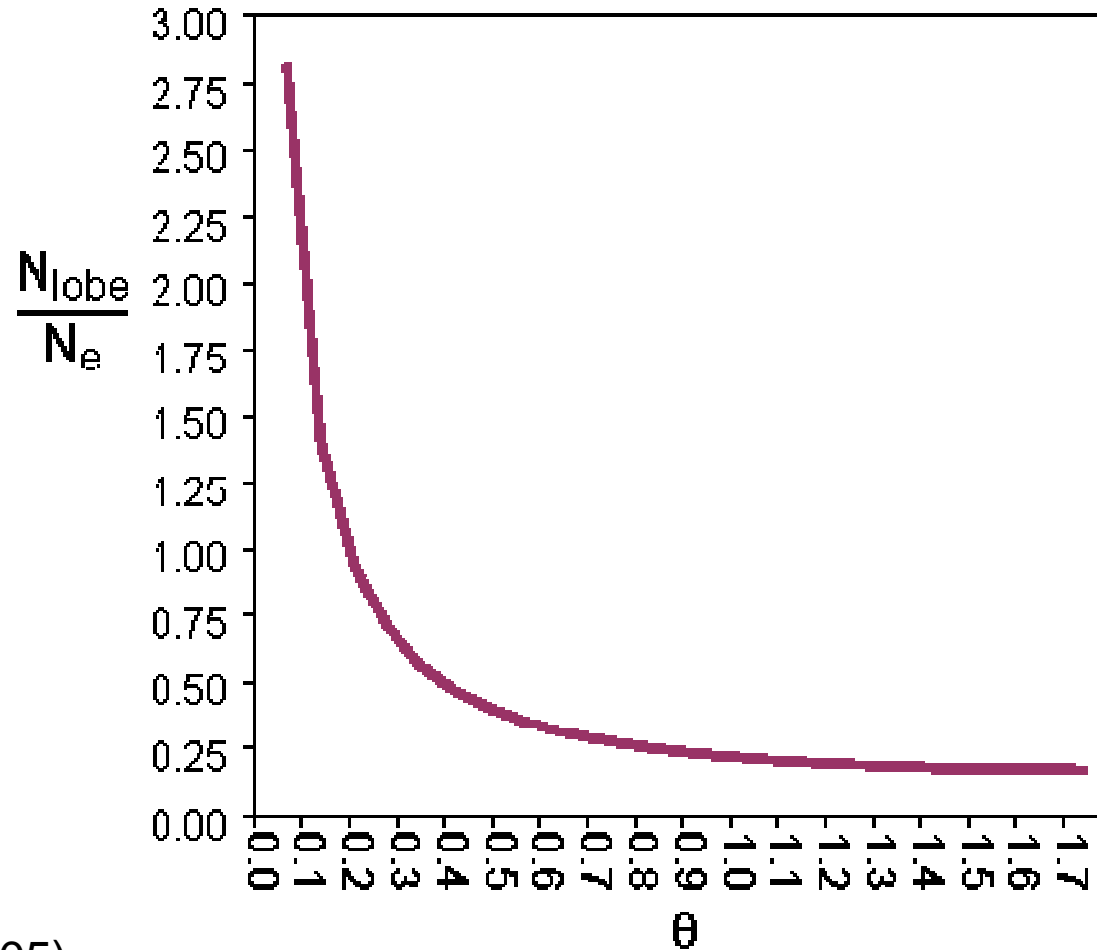
# Lobed Structure

$$N = pr$$



# Lobed Structure

$$\frac{N_{lobe}}{N_e} = \frac{r}{R_e}$$



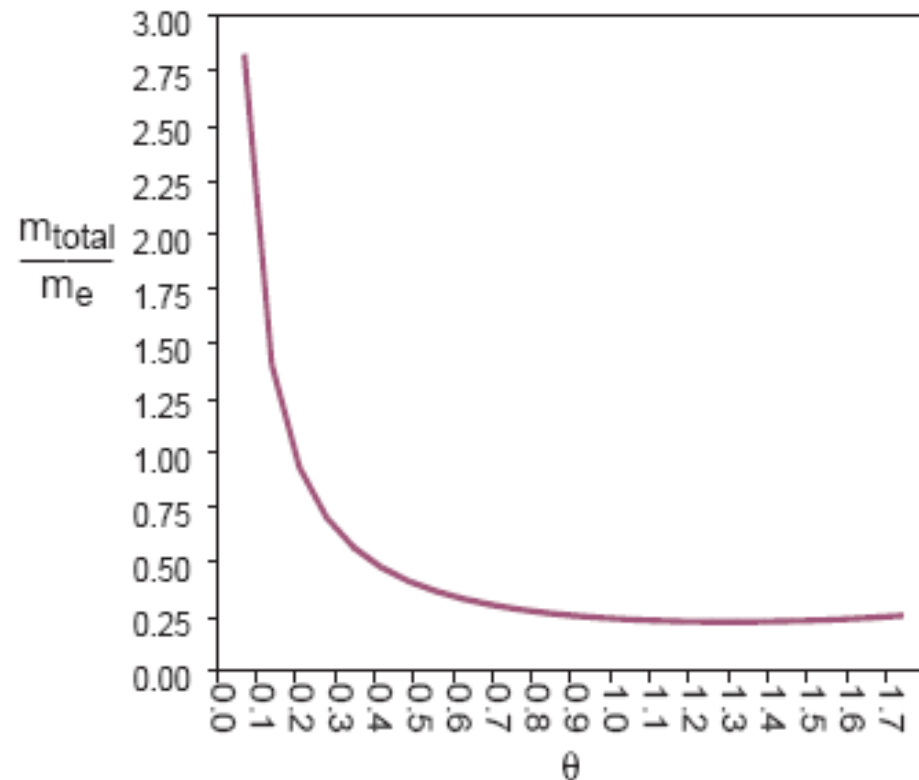
(see Lennon & Pellegrino 2005)



# Lobed Structure

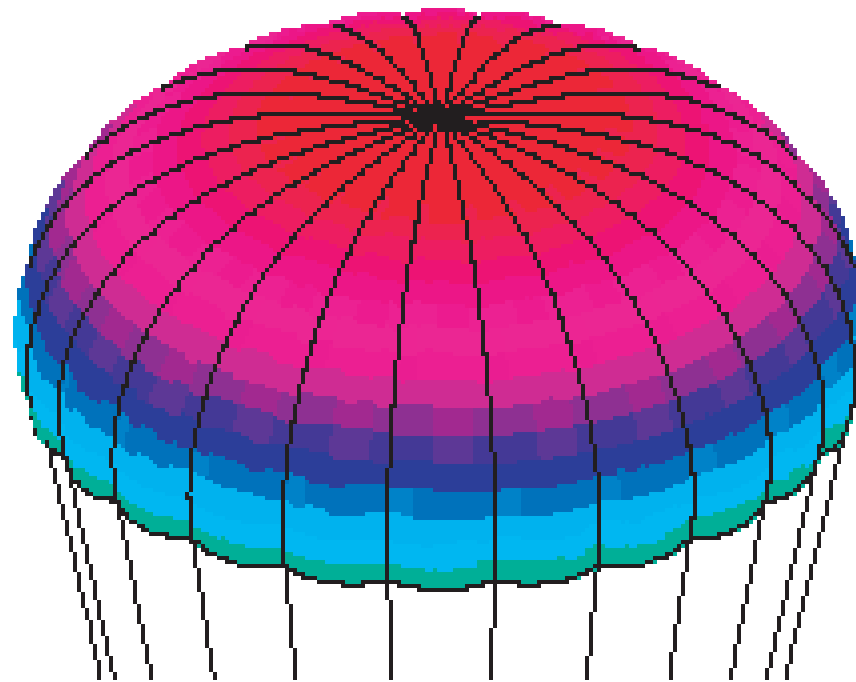
$$\frac{m_{total}}{m_e} = \left( \frac{n\theta}{\pi} \right) \left( \frac{R}{R_e^2} \right) \left( \frac{\sin\left(\frac{\pi}{n}\right)}{\sin\theta} \right)$$

mass reduction due to use of lobes to give equal stress to equivalent circle (for  $n=16$ )



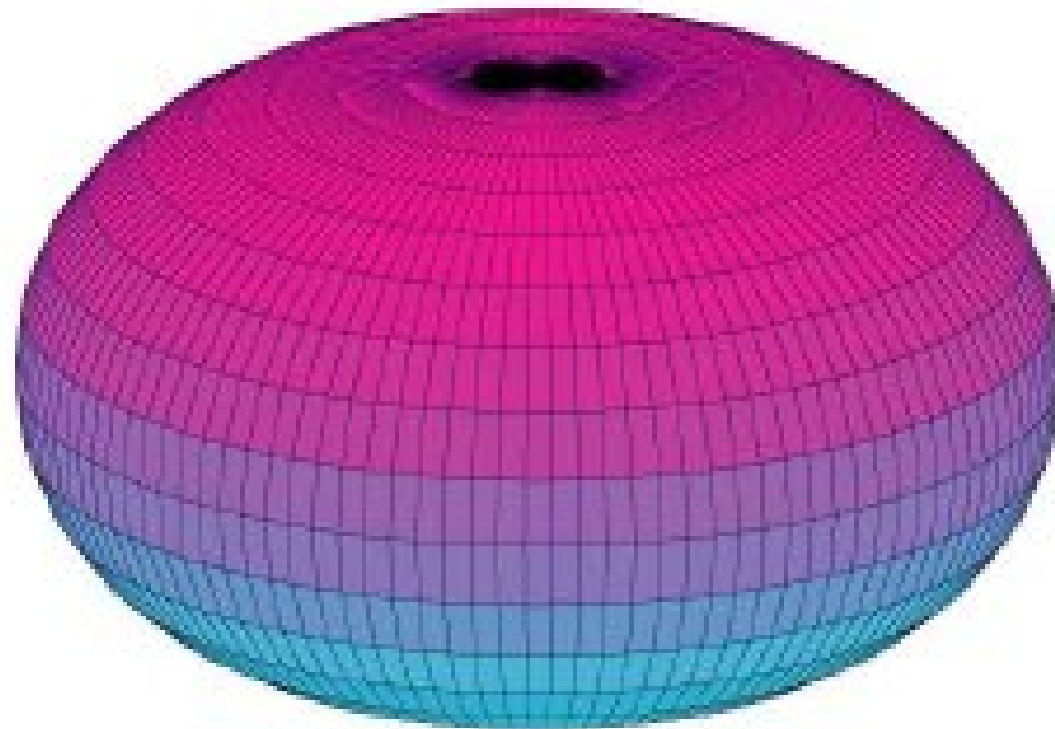
(see Lennon & Pellegrino 2005)

# Unidirectional Tension

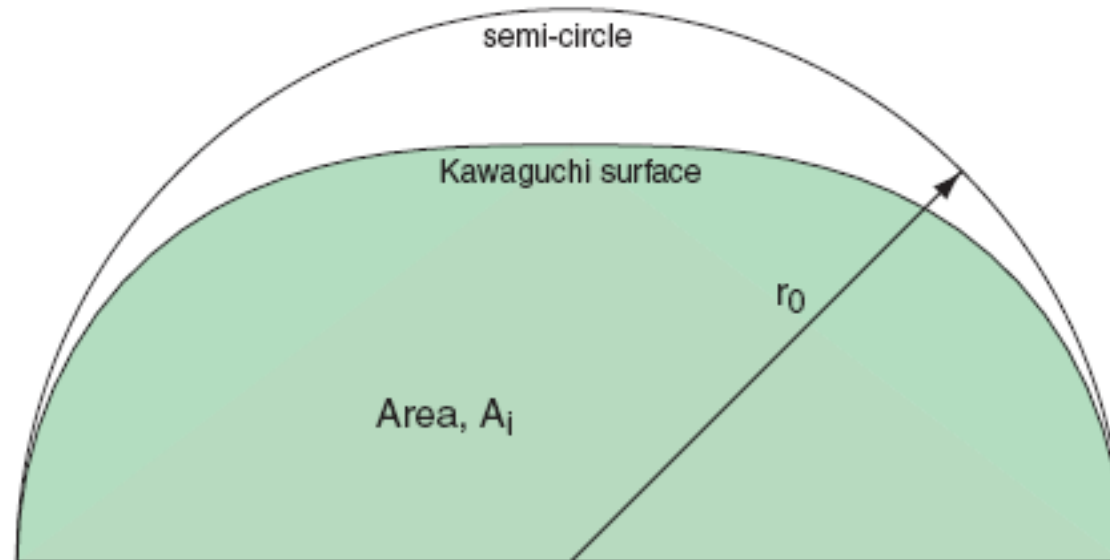


(see Taylor 1919 in Batchelor 1963)

# Unidirectional Tension

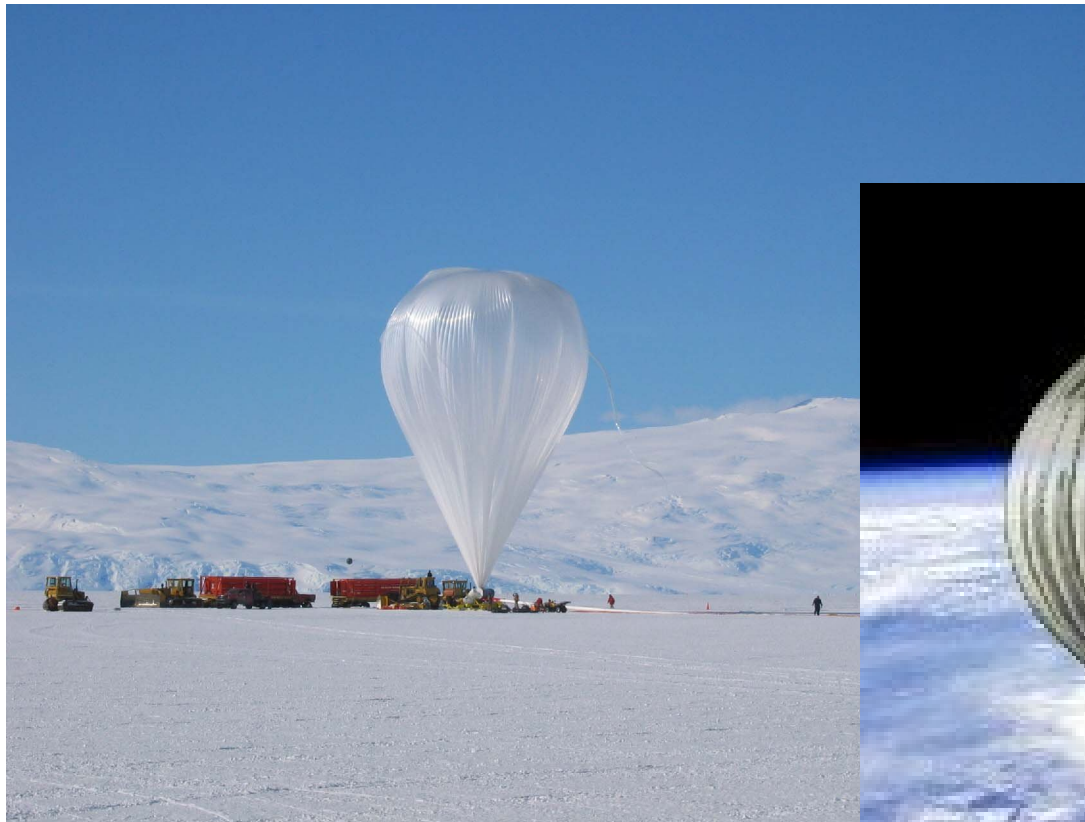


# Shallowest Possible Dome



(see Kawaguchi 1977)

# Scientific “Pumpkin” Balloons

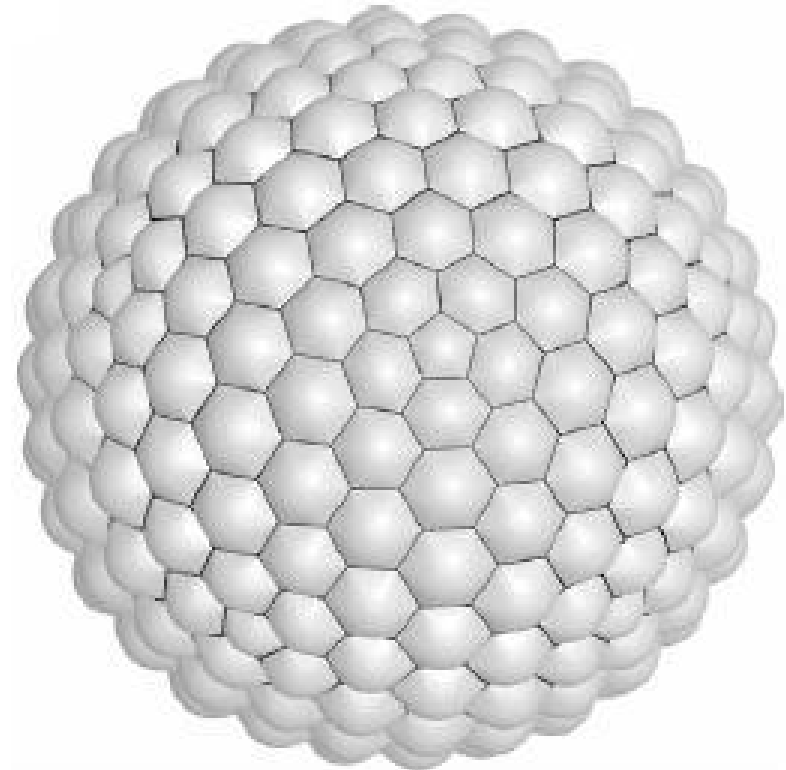


(images from NASA)

# A Note on Folds and Wrinkles



# New Ideas



(see Pagitz 2007)

# Conclusions

- Geometry has a significant effect on tension
- Manipulation of geometry can reduce tension
- Need to remember difficulty of construction
- More work to be done on geometry



# Contact Details

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